Math 30, Homework 9 on sections 4.7, 5.1, 5.2

Write all your working out and answers on your own notepaper - no need to write the questions. Please use lots of space.

It is very important that you show clearly any work you had to do to get your answers. Just writing the answer down with no work shown is usually not enough. Do all 15 questions - they are worth 2 points each. Hand in your solutions next week only.

For these first 10 questions, *check that your answers match the solutions on pages 2, 3*. If you don't get the same answers then look at your notes or the book or ask me. Only do the last five questions when you are sure you understand the first ten.

(1) Find exactly: $\sin^{-1}(1/2)$

(2) Find exactly: $\cos^{-1}(-1)$

(3) Find on your calculator, correct to 3 places: $\sin^{-1}(0.8)$

(4) Find exactly: $\cos(\cos^{-1}(1/3))$

(5) Find exactly: $\tan(\sin^{-1}(2/9))$

(6) Verify the trigonometric identity

$$\cos x \csc x = \cot x$$

(7) Verify the trigonometric identity

$$\tan(-x)\cos(x) = -\sin(x)$$

(8) Verify the trigonometric identity

$$\frac{\sin\theta}{1+\cos\theta} + \frac{-1+\cos\theta}{\sin\theta} = 0$$

(9) Use the difference formula

$$\cos(\alpha-\beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

applied to the difference $\pi/3 - \pi/4$ to compute $\cos(\pi/12)$ exactly.

(10) Use the difference formula from question 9 to verify the trigonometric identity

$$\cos(x - \pi/2) = \sin(x)$$

Five more questions. Show clearly all your working out and reasoning.

(11) Find exactly: $\cos^{-1}(\sqrt{2}/2)$

(12) Find exactly: $\sin(\cos^{-1}(6/7))$

(13) Verify the trigonometric identity

$$\csc\theta - \sin\theta = \cot\theta\cos\theta$$

(14) Verify the trigonometric identity

$$1 - \frac{\cos^2(x)}{1+\sin(x)} = \sin(x)$$

(15) Use the sum formula

$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

to verify the trigonometric identity

$$\sqrt{2}\sin(x+\pi/4) = \sin(x) + \cos(x)$$

Answers to questions (1)-(10):

(1) $\sin^{-1}(1/2) = \pi/6$

(2) $\cos^{-1}(-1) = \pi$

(3) $\sin^{-1}(0.8) \approx 0.927$

(4) $\cos(\cos^{-1}(1/3)) = 1/3$

(5) $\tan(\sin^{-1}(2/9)) = \dfrac{2}{\sqrt{77}} = \dfrac{2\sqrt{77}}{77}$

(6) Start with the left side:

$$\begin{aligned}\cos x \csc x &= \cos x \left(\frac{1}{\sin x}\right) \\ &= \frac{\cos x}{\sin x} \\ &= \cot x,\end{aligned}$$

using the reciprocal and quotient identities. So the left side equals the right side, as we wanted to show.

(7) Start with the left side:

$$\begin{aligned}\tan(-x)\cos(x) &= \frac{\sin(-x)}{\cos(-x)}\cos(x) \\ &= \frac{-\sin(x)}{\cos(x)}\cos(x) \\ &= -\sin(x),\end{aligned}$$

using the quotient and odd/even identities. So the left side equals the right side, as we wanted to show.

(8) Start with the left side:

$$\begin{aligned}\frac{\sin\theta}{1+\cos\theta} + \frac{-1+\cos\theta}{\sin\theta} &= \frac{\sin^2\theta}{(1+\cos\theta)\sin\theta} + \frac{(-1+\cos\theta)(1+\cos\theta)}{\sin\theta(1+\cos\theta)} \\ &= \frac{\sin^2\theta + \cos^2\theta - 1}{\sin\theta(1+\cos\theta)} \\ &= \frac{0}{\sin\theta(1+\cos\theta)} = 0,\end{aligned}$$

using a common denominator to add and the Pythagorean identity. So the left side equals the right side, as we wanted to show.

(9) $\cos(\pi/12) = \dfrac{\sqrt{2}+\sqrt{6}}{4}$

(10) Verify the given trigonometric identity by using the difference formula and computing $\cos(\pi/2)$ and $\sin(\pi/2)$.