Math 30, Homework 4 on sections 2.3, 2.4, 2.5

Write all your working out and answers on your own notepaper - no need to write the questions. Please use lots of space.

It is very important that you show clearly any work you had to do to get your answers. Just writing the answer down with no work shown is usually not enough. Do all 15 questions - they are worth 2 points each. Hand in your solutions next week only.

For these first 10 questions, *check that your answers match the solutions on page 3*. If you don't get the same answers then look at your notes or the book or ask me. Only do the last five questions when you are sure you understand the first ten.

(1) Identify which of these are polynomials:

- **(a)** $\dfrac{x^3+5x+3}{x-1}$
- **(b)** $4x^3-\sqrt{5}x+6/5$
- **(c)** $-9x^2+6\sqrt{x}+7$

(2) For the polynomial $f(x)=-2x^4+3x^2-9x-20$ identify:

- **(a)** its degree,
- **(b)** its leading coefficient,
- **(c)** its constant term.

(3) Use the Leading Coefficient Test to find the end behavior of

$$-3x^4+5x^3-3x^2+7x+6.$$

(4) Use long division to find the quotient and remainder for the division

$$(6x^3+7x^2+12x-5)\div(3x-1).$$

(5) Use synthetic division to find the quotient and remainder for the division

$$(5x^2-12x-8)\div(x+3).$$

(6) Use synthetic division and the Remainder Theorem to find $f(-2)$ for the polynomial

$$f(x)=3x^3-6x^2+2x-7.$$

(7) Is $x-3$ a factor of $x^4+x^3-6x^2-8x-29$?

(8) Let
$$g(x) = 5x^7 - 9x^6 + x^5 - 9x^3 + 88x^2 - 10001x - 5.$$
List all possible rational zeros of $g(x)$ according to the Rational Zeros Theorem.

(9) Let
$$f(x) = x^3 + 4x^2 - 4x - 16.$$

- **(a)** List all possible rational zeros of $f(x)$ according to the Rational Zeros Theorem.
- **(b)** Find one actual zero by using synthetic division to test the possible zeros from (a).
- **(c)** Find all remaining zeros by factoring the quotient from the zero you found in (b).
- **(d)** Write down the complete factorization of $f(x)$.

(10) Sketch the graph of $f(x)$ from question 9, making sure you show all x and y intercepts.

Five more questions. Show clearly all your working out and reasoning.

(11) Use the Leading Coefficient Test to find the end behavior of
$$-x^5 + 2x^3 - 3x^2 + 6.$$

(12) Find the quotient and remainder for the division
$$(-2x^4 + 35x^2 - 12x - 5) \div (x - 4).$$
(If using synthetic division, remember to add a 0 for the missing x^3 term.)

(13) Use synthetic division and the Remainder Theorem to find $g(5)$ for the polynomial
$$g(x) = x^4 + 1.$$

(14) Let
$$h(x) = 2x^3 - x^2 - 18x + 9.$$

- **(a)** List all possible rational zeros of $h(x)$ according to the Rational Zeros Theorem.
- **(b)** Find one actual zero by using synthetic division to test the possible zeros from (a).
- **(c)** Find all remaining zeros by factoring the quotient from the zero you found in (b).
- **(d)** Write down the complete factorization of $h(x)$.

(15) Sketch the graph of $h(x)$ from question 9, making sure you show all x and y intercepts.

Answers to questions (1)-(10):

(1) (a) No, (b) Yes, (c) No.

(2) (a) Degree is 4, (b) Leading coefficient is -2, (c) Constant term is -20.

(3) It goes down (falls) as you go to the left and to the right.

(4) The quotient is $2x^2+3x+5$ and the remainder is 0. In other words

$$(6x^3+7x^2+12x-5)\div(3x-1)=2x^2+3x+5$$

and

$$6x^3+7x^2+12x-5=(3x-1)(2x^2+3x+5).$$

(5) The quotient is $5x-27$ and the remainder is 73. In other words

$$(5x^2-12x-8)\div(x+3)=5x-27+\frac{73}{x+3}$$

and

$$5x^2-12x-8=(x+3)(5x-27)+73.$$

(6) $f(-2)=-59$

(7) No (because the remainder after division is 1).

(8) There are six possibilities: $\pm1, \pm5, \pm1/5$

(9) (a) The possible rational zeros are: $\pm1, \pm2, \pm4, \pm8, \pm16$
(b) We see that 2 is an actual zero (also -2 or -4)
(c) All the zeros are: $2, -2, -4$
(d) The complete factorization of $f(x)$ is $(x-2)(x+2)(x+4)$.

(10) Draw a smooth curve passing through $-4, -2, 2$ on the x axis and -16 on the y axis. Its end behavior falls to the left and rises to the right.