Math 30, Homework 3 on sections 1.8, 2.2

Write all your working out and answers on your own notepaper - no need to write the questions. Please use lots of space.

It is very important that you show clearly any work you had to do to get your answers. Just writing the answer down with no work shown is usually not enough. Do all 15 questions - they are worth 2 points each. Hand in your solutions next week only.

For these first 10 questions, *check that your answers match the solutions on pages 3, 4*. If you don't get the same answers then look at your notes or the book or ask me. Only do the last five questions when you are sure you understand the first ten.

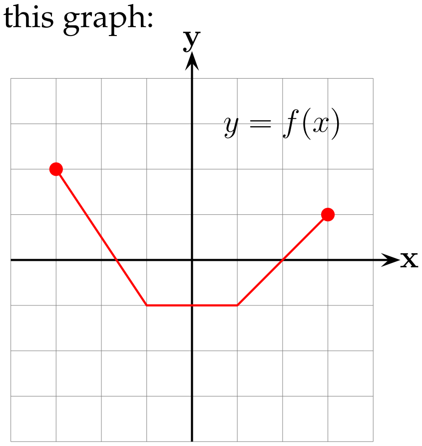
(1) Let $f(x) = 4x + 1$ and $g(x) = \dfrac{x-1}{4}$.

- **(a)** Compute $f(g(x))$.
- **(b)** Compute $g(f(x))$.
- **(c)** Say why these functions are inverses of each other (or why not).

(2) Let $f(x) = 3x - 2$ and $g(x) = \dfrac{x+3}{2}$.

- **(a)** Compute $f(g(x))$.
- **(b)** Compute $g(f(x))$.
- **(c)** Say why these functions are inverses of each other (or why not).

(3) Suppose the function $f(x)$ has this graph:



Does $f(x)$ have an inverse? Explain.

(4) Find the inverses of these functions:

(a) $g(x) = -x$

(b) $h(x) = \sqrt{x}$

(c) $k(x) = (x+2)^3$

(5) Let $g(x) = \dfrac{2x+1}{x-3}$.

(a) Find the domain of $g(x)$.

(b) Find its inverse, $g^{-1}(x)$.

(c) Find the range of $g(x)$.

(6) Draw the graph of $f(x) = -2(x-2)^2 + 8$.

(7) The graph of $f(x) = 2x^2 - 8x + 3$ is a parabola. Find the coordinates of its vertex.

(8) Find the vertex and intercepts of the graph of $g(x) = x^2 - 4$. Then use these to sketch the graph.

(9) Suppose a quadratic function has the following graph: a parabola opening downwards with vertex at $(-3, 2)$.

(a) Find the domain of this function.

(b) Find the range.

(c) Does it have a local maximum or minimum?

(10) Find the domain and range of this function: $h(x) = 2x^2 + 4x - 1$.

Five more questions. Show clearly all your working out and reasoning.

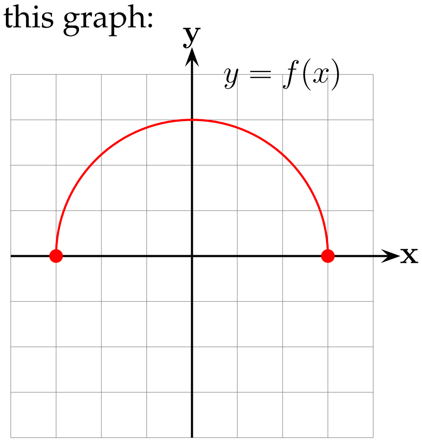
(11) Let $f(x) = \dfrac{x}{2} + 1$ and $g(x) = 2x - 2$.

(a) Compute $f(g(x))$.

(b) Compute $g(f(x))$.

(c) Say why these functions are inverses of each other (or why not).

(12) Suppose the function $f(x)$ has this graph:



Does $f(x)$ have an inverse? Explain.

(13) Let $g(x) = \dfrac{-5x+3}{2x+1}$.

(a) Find the domain of $g(x)$.

(b) Find its inverse, $g^{-1}(x)$.

(c) Find the range of $g(x)$.

(14) Find the vertex and intercepts of the graph of $g(x) = -3x^2 + 6x + 9$. Then use these to sketch the graph.

(15) Find the domain and range of this function: $q(x) = x^2 - 6x + 4$.

Answers to questions (1)-(10):

(1) (a) $f(g(x)) = x,$ (b) $g(f(x)) = x,$
(c) The relations in (a) and (b) mean f and g are inverses of each other.

(2) (a) $f(g(x)) = \dfrac{3x-9}{2},$ (b) $g(f(x)) = \dfrac{3x+1}{2},$
(c) The results in (a) and (b) mean f and g are not inverses of each other.

(3) This graph fails the horizontal line test. That means f is not one-to-one. Therefore f does not have an inverse.

(4) (a) $g^{-1}(x) = -x,$ (b) $h^{-1}(x) = x^2,$ (c) $k^{-1}(x) = \sqrt[3]{x} - 2$

(5) (a) domain $g = (-\infty, 3) \cup (3, \infty),$ (b) $g^{-1}(x) = \dfrac{3x+1}{x-2},$
(c) range $g = (-\infty, 2) \cup (2, \infty)$

(6) One way to see this graph is to use these transformations of the graph of $y = x^2$: first move it right 2 units, then reflect it through the x-axis, stretch it by a factor of 2 and finally move it up 8 units. Your picture should be an "n" shaped parabola with top point at $(2, 8)$.

(7) Vertex is at $(2,-5)$

(8) The vertex is at $(0,-4)$ which is also the y-intercept. The x-intercepts are at $(-2,0)$ and $(2,0)$. Draw a smooth ″U″ shaped parabola through these points.

(9) (a) domain is $(-\infty,\infty)$, (b) range is $(-\infty,2]$, (b) it has a local maximum

(10) The domain is $(-\infty,\infty)$ and the range is $[-3,\infty)$.