Mth 21, Homework 5 on section 7.4, 7.5

Due by Wed, Oct 11.

Please use lots of space and explain your answers, showing clearly any work you had to do. Each question is worth 3 points.

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(1) Use the sieve of Eratosthenes to find all the prime numbers between 1 and 60.

(Hint: make a grid $1-10$, $11-20$, ..., $51-60$. Cross out 1 and repeat the following procedure: circle the next number and cross out all its multiples. You don't need to remember which numbers are prime - the sieve finds them all for you.)

(2) Give the prime factorization of 630.

(3) Is the number 541 prime or composite? Explain.

(Hint: see if any of the primes up to $\sqrt{541}$ divide it evenly. If one does it's composite. If none do it's prime.)

(4) Check if the following numbers are abundant, perfect or deficient:

(a) 32

(b) 53

(c) 100

(Hint: add up all the proper factors of each number and compare with the number.)

(5) Compute the fourth perfect number by using Euclid's formula

$$2^{n-1}(2^n-1)$$

when $n=7$. (This formula is 2300 years old.)

(6) RSA encryption is a common way for things like credit card numbers to be kept safe online. It works by choosing two very large prime numbers p and q. This is the private key and kept secret. The product pq is called the public key and made public. The public key is used to encrypt (hide) the data and the private key is needed to decrypt (reveal) the data.

Suppose 323 is a public key. Break the encryption to get the private key and the steal the data.

(In reality the public key would have hundreds of digits and not be so easy to break! See p. 520 of the book for more information.)

(7) The Fibonacci numbers are a sequence of numbers that start

$$F_1 = 1, \quad F_2 = 1, \quad F_3 = 2, \quad F_4 = 3, \ldots$$

with the next number the sum of the previous two. Compute this sequence up to F_{15}. (Hint: you should get $F_{15} = 610$.)

(8) Use Binet's formula

$$F_n = \frac{\phi^n - 1/(-\phi)^n}{\sqrt{5}}$$

(only about 300 years old), to compute the sixteenth Fibonacci number as follows:

(a) On your calculator compute the golden ratio $\phi = (1 + \sqrt{5})/2$

(b) Raise this number to the power 16

(c) Divide your answer by $\sqrt{5}$ and round to the closest integer. That is F_{16}. (The $-1/(-\phi)^n$ part of Binet's formula may be ignored as it is very small.)

(9) Check that your answer in Question 8 is the sum of F_{14} and F_{15} in Question 7.

If you get stuck on a question or aren't sure if you understand it:

- Go over the relevant class notes and section in the textbook.
- Check if you get the right answer for a similar odd-numbered question in the textbook (answers at the back of the book).
- Ask me about it after class.
- Come to my office hours: Mon 11:30 - 12:30, Wed 11:30 - 12:30 in CP 317.
- Go to the Math Tutorial Lab in-person in CP 303 or online.