CSI 35, Homework 6 on section 5.3

Due by Wed, Oct 26.

Here are nine questions for you to try with one from the book. Write all your working out and answers on your own notepaper - no need to write the questions. Please use lots of space.

It is very important that you show clearly any work you had to do to get your answers. Just writing the answer down with no work shown is usually not enough.

---

(1) Define the function $f(n)$ recursively with

$$\begin{aligned} \text{Basis step:} &\quad f(0) = 2 \\ \text{Recursive step:} &\quad f(n+1) = f(n)^2 - 3. \end{aligned}$$

Show all your steps to compute $f(4)$.

(2) Define the sequence b_n recursively with

$$\begin{aligned} \text{Basis step:} &\quad b_0 = 1,\ b_1 = 3 \\ \text{Recursive step:} &\quad b_{n+1} = b_n - 2b_{n-1}. \end{aligned}$$

Show that $b_5 = 3$.

(3) Question 7, parts (a), (c) only, on page 358.

(4) Recall the recursive definition of the Fibonacci numbers from the notes. Use induction to prove that

$$f_1 + f_3 + \cdots + f_{2n-1} = f_{2n}$$

for all $n \geqslant 1$.

(5) What are the strings in Σ^* if the alphabet $\Sigma = \{8\}$?

(6) Define the set T recursively with

$$\begin{aligned} \text{Basis step:} &\quad (0,0) \in T \\ \text{Recursive step:} &\quad \text{if } (a,b) \in T \text{ then } (a+1, b-2) \in T \text{ and } (a-1, b+1) \in T. \end{aligned}$$

(a) Give five different elements of T.

(b) Is $(0, -3)$ in T? Explain why or why not.

(7) Define the set U recursively with

$$\begin{aligned} \text{Basis step:} &\quad (0,0) \in U \\ \text{Recursive step:} &\quad \text{if } (a,b) \in U \text{ then } (a+1, b-1) \in U \text{ and } (a-4, b+4) \in U. \end{aligned}$$

Use structural induction to prove that if $(a,b) \in U$ then $a + b = 0$.

(8) Draw an example of a full binary tree with exactly 13 vertices.

(9) Our first example of a recursively defined function was $g(n)$ with

$$\begin{aligned} \text{Basis step:} \quad & g(0) = 1 \\ \text{Recursive step:} \quad & g(n+1) = 2g(n) + n - 1. \end{aligned}$$

Use induction (regular induction or structural induction) to prove that

$$g(n) = 2^n - n \qquad \text{for} \qquad n \geqslant 0.$$

If you understand the homework questions then you will be able to do the exam questions. You can also try the other questions listed on the syllabus to get extra practice. For any difficulties with the homework, please email me, come to my office hours or try the Math Tutoring Lab.