CSI 35, Homework 5 on sections 5.1, 5.2
Due by Wed, Oct 19.

Here are seven questions for you to try with some from the book. Write all your working out and answers on your own notepaper - no need to write the questions. Please use lots of space.

It is very important that you show clearly any work you had to do to get your answers. Just writing the answer down with no work shown is usually not enough.

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(1) Use induction to prove that

$$1 + 4 + 4^2 + \cdots + 4^n = \frac{4^{n+1} - 1}{3}$$

for all $n \geqslant 1$.

(2) Use induction to prove that 2 divides $n^2 + 5n$ for all $n \geqslant 0$.

(3) Question 18 on page 330. The notation $n!$ means $1 \cdot 2 \cdot 3 \cdots (n-1)n$ which is the product of the first n positive integers.

(4) Question 49 on page 331.

(5) In the usual induction, the inductive hypothesis is that $P(k)$ is true. Explain what the inductive hypothesis is for strong induction.

(6) Question 4 on page 341.

(7) [Extra credit] Question 12 on page 342. In this question we want to prove that every positive integer n can be written as a sum of distinct powers of 2. For example when $n = 101$,
$$101 = 2^0 + 2^2 + 2^5 + 2^6.$$

For the inductive step we want to relate $k + 1$ to a smaller number that we know is a sum of powers of 2. (If $k + 1$ is even then $(k + 1)/2$ must be an integer and if $k + 1$ is odd then $(k + 1 - 1)/2$ must be an integer.)

If you understand the homework questions then you will be able to do the exam questions. You can also try the other questions listed on the syllabus to get extra practice. For any difficulties with the homework, please email me, come to my office hours or try the Math Tutoring Lab.