CSI 35, Homework 4 on section 5.1

Due by Wed, Oct 12.

Here are six questions for you to try. Write all your working out and answers on your own notepaper - no need to write the questions. Please use lots of space.

It is very important that you show clearly any work you had to do to get your answers. Just writing the answer down with no work shown is usually not enough.

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(1) A *proposition* is a statement that is either true or false (and not both).

- **(a)** Why is the statement "The next coin I toss will land heads" not a proposition?
- **(b)** Is the proposition "$x^2 - x + 3 = 1$ when $x = 2$" true or false?
- **(c)** Is the proposition "$(-1)^{1000019} = -1$" true or false?

(2) Suppose, in a large hot star, when 4 hydrodgen atoms combine into one helium atom then the energy produced always causes 4 more hydrogen atoms to combine into a helium atom. If this star reaches a temperature to cause the first 4 hydrodgen atoms to combine into a helium atom, what happens after that?

(3) Prove the formula

$$\underbrace{(-7) + (-7) + \cdots + (-7)}_{n \text{ of these}} = -7n$$

for $n \geqslant 1$ by induction using these steps:

- **(a)** Identify the proposition we want to prove and write "$P(n)$ says ....".
- **(b)** Check the *basis step*.
- **(c)** Complete the *inductive step* as follows: write down what $P(k)$ says, assume it's true, and then use this to prove that $P(k+1)$ must also be true.
- **(d)** Write your conclusion: "So by mathematical induction $P(n)$ is true for all $n \geqslant 1$".

(4) Use the same four step as the last question to prove by induction that

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

is true for all $n \geqslant 1$.

(5) Consider the sum

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{n(n+1)}.$$

(a) When $n = 1$ the sum is just

$$\frac{1}{1 \cdot 2} = \frac{1}{2}.$$

Compute the fraction the sum equal to when $n = 2$:

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} =$$

(b) Compute the fraction the sum equal to when $n = 3$:

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} =$$

(c) Use parts (a) and (b) to conjecture (guess) what the formula for the sum should be for any n.

(6) Use induction to prove your conjectured formula from the last question.

If you understand the homework questions then you will be able to do the exam questions. You can also try the other questions listed on the syllabus to get extra practice. For any difficulties with the homework, please email me, come to my office hours or try the Math Tutoring Lab.