**Math 32, Practice Final Exam**

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As shown below, the questions on the final will be similar to test and homework questions. This practice final is to give you an idea of what to expect and help us review. The actual final exam is on Wed, Dec 20, 2 – 4:50 pm in CP 309 (room next door).

On the final you will be asked to do any 15 of the 17 questions in 2 hours and 50 minutes. They are worth 6 points each. To get all 6 points it is very important that you show clearly all your working out and reasoning. Some formulas for inverse trigonometric functions, areas, arc lengths, and conic sections are given on the last page.

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**(1) Volume.** [See tests 1,2 and homeworks 2, 3]

Find the volume of the solid obtained by rotating the region bounded by the following curves about the horizontal line $y = -3$:

$$y = x^3, \quad y = 1, \quad x = 2.$$

**(2) Derivatives.** [See test 3 and homeworks 3, 4, 5]

Differentiate:

$$(\mathbf{a}) \quad \tan(\log_4 t) \qquad\qquad (\mathbf{b}) \quad \sqrt{1+e^{x^2}}$$

**(3) Logarithmic differentiation.** [See test 3 and homework 5]

Use logarithmic differentiation to compute $f'(x)$ for

$$f(x) = \frac{\sqrt{x}\cdot e^{2x}}{(x+1)^3}.$$

**(4) Inverse trig functions and their derivatives.** [See test 3 and homework 5]

Give the domain of $\sin^{-1}(x)$. Then find the derivative of: $\tan^{-1}(\sin^{-1}(x))$

**(5) Limits and l'Hospital's Rule.** [See tests 2, 3 and homework 6]

Compute:

$$(\mathbf{a}) \quad \lim_{x\to 0} \frac{\cos(x)-1}{x^2} \qquad\qquad (\mathbf{b}) \quad \lim_{x\to\infty} x^{1/x}$$

**(6) Integration by parts.** [See test 4 and homework 7]

Evaluate:

$$(\mathbf{a}) \quad \int xe^x\,dx \qquad\qquad (\mathbf{b}) \quad \int \arcsin(x)\,dx$$

**(7) Trigonometric integrals.** [See test 4 and homework 7]

Calculate: $\displaystyle\int_0^{\pi/2} \sin^{10}(x)\cos^3(x)\,dx$

**(8) Trigonometric substitution.** [See test 4 and homeworks 7, 8]

Find: $\displaystyle\int \frac{x^3}{\sqrt{x^2+9}}\,dx$

**(9) Integrating rational functions.** [See test 4 and homework 8]

Calculate: $\displaystyle\int \frac{3x^2-7x+27}{x^3+9x}\,dx$

**(10) Integration.** [See test 4 and homeworks 8, 9]

Find the right strategies of integration to compute:

$$(\mathbf{a})\ \int \ln(1+x^2)\,dx \qquad\qquad (\mathbf{b})\ \int \frac{dx}{1+\sqrt{x}}$$

**(11) Improper integrals.** [See test 4 and homework 9]

Are the following two integrals convergent or divergent? Compute them if they are convergent:

$$(\mathbf{a})\ \int_3^\infty \frac{1}{x^{2/3}}\,dx \qquad\qquad (\mathbf{b})\ \int_{-\infty}^0 3^x\,dx$$

**(12) Arclength.** [See test 4 and homework 9]

Find the length of the curve $y = 2x^{3/2}$ for $0 \leqslant x \leqslant 11$.

**(13) Area of surface of revolution.** [See test 5 and homework 10]

Find the area obtained by rotating this curve about the $x$-axis: $y = x^3,\ 0 \leqslant x \leqslant 2$

**(14) Graphing a polar curve.** [See test 5 and homework 10]

Graph the polar curve $r = \cos(2\theta)$.

**(15) Area of a region bounded by a polar curve.** [See test 5 and homework 10]

Find the area of one leaf of the curve in question 14.

**(16) Conic sections.** [See test 5 and homeworks 10, 11]

For the conic section

$$\frac{x^2}{15^2} - \frac{y^2}{8^2} = 1$$

find its foci, vertices, eccentricity and any asymptotes. Then sketch it.

**(17) Conic sections in polar coordinates.** [See test 5 and homework 11]

Find a polar equation for a conic section with eccentricity $2/3$ and perihelion distance $5$. Then find the aphelion distance.

## Formulas

- We have
$$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}, \qquad \frac{d}{dx}\cos^{-1}(x) = \frac{-1}{\sqrt{1-x^2}}, \qquad \frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}.$$

- The length of the curve $y = f(x)$ for $a \leqslant x \leqslant b$ is
$$\int_a^b \sqrt{1+(f'(x))^2}\,dx.$$

- If the curve $y = f(x)$ for $a \leqslant x \leqslant b$ is rotated about the $x$-axis then it makes a surface with area
$$\int_a^b 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx.$$

- The area of the polar region bounded by $r = f(\theta)$ with $a \leqslant \theta \leqslant b$ is
$$\frac{1}{2}\int_a^b f(\theta)^2\,d\theta.$$

- The parabola with focus at $(0,p)$, vertex at $(0,0)$ and directrix $y = -p$ has equation
$$4py = x^2.$$

- The ellipse with equation
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
for $a \geqslant b > 0$ has vertices at $(\pm a, 0)$ and foci at $(\pm c, 0)$ where $c^2 = a^2 - b^2$. Its eccentricity is $c/a$.

- The hyperbola with equation
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$
has vertices at $(\pm a, 0)$ and foci at $(\pm c, 0)$ where $c^2 = a^2 + b^2$. Its eccentricity is $c/a$ and its asymptotes are $y = \pm(b/a)x$.

- A polar equation for a conic section with a focus at the origin and eccentricity $e > 0$ is
$$r = \frac{ed}{1 + e\cos(\theta)}.$$