Math 32, Homework 9 on sections 7.5, 7.8, 8.1

Do these 11 questions and *check that your answers match the solutions on page 2*. They will not be collected, but similar questions could appear on the next quiz.

(1) Compute:

$$\int e^{\sqrt{t}}\,dt$$

(2) Find:

$$\int \frac{d\theta}{1+\cos^2\theta}$$

This is a difficult one. Try first rewriting the integrand as $\dfrac{\sec^2\theta}{\sec^2\theta+1}$ and then use a trigonometric identity.

(3) Is this integral convergent or divergent? Evaluate it if it is convergent:

$$\int_1^\infty \frac{1}{x^5}\,dx$$

(4) Is this integral convergent or divergent? Evaluate it if it is convergent:

$$\int_3^\infty \frac{1}{x^{2/3}}\,dx$$

(5) Is this integral convergent or divergent? Compute it if it is convergent:

$$\int_e^\infty \frac{dx}{x(\ln x)^2}$$

(6) Is this integral convergent or divergent? Compute it if it is convergent:

$$\int_{-\infty}^0 3^x\,dx$$

(7) Is this integral convergent or divergent? Calculate it if it is convergent:

$$\int_0^9 \frac{dx}{\sqrt{x}}$$

(8) Is this integral convergent or divergent? Calculate it if it is convergent:

$$\int_{-2}^4 \frac{dx}{(x-1)^3}$$

(9) Set up an integral to give the length of the curve $y = \tan x$ for $-\pi/4 \leqslant x \leqslant \pi/4$. Don't try to compute the integral (probably impossible).

(10) Find the length of the curve $y = 2x^{3/2}$ for $0 \leqslant x \leqslant 11$.

(11) Find the length of the curve $y = \dfrac{x^2}{4} - \dfrac{\ln x}{2}$ for $1 \leqslant x \leqslant 2$. For this it is useful to notice that:

$$\frac{x^2}{4} + \frac{1}{2} + \frac{1}{4x^2} = \left(\frac{x}{2} + \frac{1}{2x}\right)^2$$

You can also try questions from sections 7.5, 7.8, 8.1 in the book listed on the syllabus.

Answers to questions (1)-(11):

(1) $2\sqrt{t}e^{\sqrt{t}} - 2e^{\sqrt{t}} + C$

(2) $\dfrac{1}{\sqrt{2}} \tan^{-1}\left(\dfrac{\tan(\theta)}{\sqrt{2}}\right) + C$

(3) This integral is convergent and equals $1/4$.

(4) This integral is divergent.

(5) This integral is convergent and equals 1.

(6) This integral is convergent and equals $1/\ln(3)$.

(7) This integral is convergent and equals 6.

(8) This integral is divergent.

(9) $\displaystyle\int_{-\pi/4}^{\pi/4} \sqrt{1 + \sec^4(x)}\, dx$

(10) 74

(11) $\dfrac{3}{4} + \dfrac{\ln 2}{2}$