Math 32, Homework 3 on sections 5.3, 6.1, 6.2

Do these 10 questions and *check that your answers match the solutions on page 2*. They will not be collected, but similar questions could appear on the next quiz.

(1) Sketch the region enclosed by the following curves and lines

$$y = x^2, \quad y = 0, \quad x = 1, \quad x = 2.$$

Then use the method of cylindrical shells to find the volume of the solid obtained by rotating this region about the y-axis.

(2) Sketch the region enclosed by the following curve and line

$$y = 4x - x^2, \quad y = x.$$

Then use the method of cylindrical shells to set up the integral to find the volume of the solid obtained by rotating this region about the y-axis. (No need to compute the integral.)

(3) Is the function $f(x) = 2 + \cos(x)$ one-to-one? Is it one-to-one on the domain $0 \leqslant x \leqslant \pi$?

(4) Let $g(x) = 3x + 1$. Find $g^{-1}(x)$ and graph both g and its inverse together.

(5) Let

$$f(x) = \frac{x+1}{2x-3}.$$

Find the inverse of f. Then give the domain and range of f and the domain and range of f^{-1}.

(6) (a) Use the formula you found in Q5 to compute $(f^{-1})'(1)$. (b) Use the formula

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

to compute $(f^{-1})'(1)$.

(7) Graph the exponential function $h(x) = (0.4)^x$. Give the domain and range of h and state whether it is increasing or decreasing. Also find $\lim_{x\to\infty} h(x)$ and $\lim_{x\to-\infty} h(x)$.

(8) Find:

$$(a) \quad \lim_{x\to\infty} \frac{2e^x - 1}{e^x + 1}, \qquad (b) \quad \lim_{x\to\infty} e^{-x}\sin(x).$$

(9) Differentiate:

$$(a) \quad e^{4\tan(x)}, \qquad (b) \quad e^{x/(x-1)}.$$

(10) Differentiate:

$$(a) \quad \sqrt{1+e^{x^2}}, \qquad (b) \quad 3x^2 e^{\cos(x)}.$$

You can also try questions from sections 5.3, 6.1, 6.2 in the book listed on the syllabus.

Answers to questions (1)-(10):

(1) The volume is $15\pi/2$.

(2) The integral giving the volume is

$$2\pi \int_0^3 (3x^2 - x^3)\, dx.$$

(3) With the horizontal line test, $f(x)$ is not one-to-one. But it is one-to-one on the domain $0 \leqslant x \leqslant \pi$.

(4) $g^{-1}(x) = (x-1)/3$. The graphs of g and g^{-1} are symmetric about the line $y = x$.

(5) We have

$$f^{-1}(x) = \frac{3x+1}{2x-1}$$

and

$$\text{domain of } f = \mathbb{R}/\{3/2\}, \quad \text{range of } f = \mathbb{R}/\{1/2\}$$
$$\text{domain of } f^{-1} = \mathbb{R}/\{1/2\}, \quad \text{range of } f^{-1} = \mathbb{R}/\{3/2\}.$$

(6) With both methods $(f^{-1})'(1) = -5$.

(7) We have

$$\text{domain of } h = \mathbb{R}, \quad \text{range of } h = (0, \infty).$$

This function is decreasing and $\lim_{x\to\infty} h(x) = 0$, $\lim_{x\to-\infty} h(x) = \infty$.

(8) We have

$$(a) \quad \lim_{x\to\infty} \frac{2e^x - 1}{e^x + 1} = 2, \qquad (b) \quad \lim_{x\to\infty} e^{-x}\sin(x) = 0.$$

(9) We have

$$(a) \quad \frac{d}{dx} e^{4\tan(x)} = 4\sec^2(x) e^{4\tan(x)}, \qquad (b) \quad \frac{d}{dx} e^{x/(x-1)} = \frac{-1}{(x-1)^2} e^{x/(x-1)}.$$

(10) We have

$$(a) \quad \frac{d}{dx}\sqrt{1+e^{x^2}} = \frac{x e^{x^2}}{\sqrt{1+e^{x^2}}}, \qquad (b) \quad \frac{d}{dx} 3x^2 e^{\cos(x)} = 3x(2 - x\sin(x)) e^{\cos(x)}.$$