Math 32, Homework 10 on sections 8.2, 10.3, 10.4, 10.5

Write all your working out and answers on your own notepaper - no need to write the questions. Please use lots of space.

It is very important that you show clearly any work you had to do to get your answers. Just writing the answer down with no work shown is not enough. All 15 questions are worth 2 points each. Hand in your solutions next week only.

Do these first 10 questions and *check that your answers match the solutions on page 2*. If you don't get the same answers then look at your notes or the book or ask me. Only do the last five questions when you are sure you understand the first ten.

(1) Find the area of the surface of revolution obtained by rotating this line about the x-axis: $y = x/2,\ 0 \leqslant x \leqslant 3$

(2) Find the area obtained by rotating this curve about the x-axis: $y = x^3,\ 0 \leqslant x \leqslant 2$

(3) Plot these three points, given in polar coordinates, and find their Cartesian coordinates: $A(2, \pi/2),\ B(-1, \pi/4),\ C(2, -\pi/3)$

(4) Plot these three points, given in Cartesian coordinates, and find their polar coordinates: $P(-2, 0),\ Q(-3, 3),\ R(0, 0)$

(5) Sketch the graph of the polar curve: $r = \cos(3\theta)$

(6) Sketch the graph of the polar curve: $r = 2 + \cos(\theta)$

(7) Find the area of the polar curve: $r = 2\cos(\theta),\ 0 \leqslant \theta \leqslant \pi$

(8) Find the length of the polar curve: $r = \theta^2,\ 0 \leqslant \theta \leqslant 2\pi$

(9) The equation $y = 8x^2$ is the equation of a parabola. Find the focus and directrix and graph these along with the parabola.

(10) Find the equation of the ellipse with foci $(\pm 2, 0)$ and vertices $(\pm 3, 0)$. Then graph this ellipse, including the foci and showing where the ellipse crosses the axes.

Five more questions. Show clearly all your working out and reasoning.

(11) Find the area obtained by rotating this curve about the x-axis: $y = \cos(x),\ 0 \leqslant x \leqslant \pi/2$. For this question it might be useful to remember that

$$\int \sec^3(\theta)\, d\theta = \frac{1}{2}\Big(\sec(\theta)\tan(\theta) + \ln\big|\sec(\theta) + \tan(\theta)\big|\Big).$$

(12) Sketch the graph of the polar curve: $r = 1 + 2\cos(\theta)$

(13) Find the area of one leaf of the polar curve from question 5: $r = \cos(3\theta)$

(14) Find the equation of a parabola with focus $(0, -3)$ and directrix $y = 3$.

(15) Compute the foci of the ellipse: $x^2/9 + y^2 = 1$

You can also try other questions from sections 8.2, 10.3, 10.4, 10.5 in the book listed on the syllabus.

Answers to questions (1)-(10):

(1) $\dfrac{9\sqrt{5}\pi}{4}$

(2) $\dfrac{\pi}{27}(145^{3/2} - 1)$

(3) The Cartesian coordinates are $(0, 2)$ for A, $(-\sqrt{2}/2, \sqrt{2}/2)$ for B and $(1, -\sqrt{3})$ for C. Plot these points.

(4) Possible polar coordinates are $(2, \pi)$ for P, $(3\sqrt{2}, 3\pi/2)$ for Q and $(0, 0)$ for R. Plot these points.

(5) This 3-leaved rose is shown in graph II in Q54 on p. 707 of the text.

(6) This curve is shown at the bottom right on p. 712 of the text.

(7) π

(8) $\dfrac{8}{3}((\pi^2 + 1)^{3/2} - 1)$

(9) The focus is at $(0, 2)$ and the equation of the directrix line is $y = -2$. Graph these with the parabola.

(10) The equation is $\dfrac{x^2}{9} + \dfrac{y^2}{5} = 1$. This ellipse crosses the x-axis at $x = \pm 3$ and the y-axis at $y = \pm\sqrt{5}$.