Math 32, Homework 1 reviewing chapters 1-4

Write all your working out and answers on your own notepaper - no need to write the questions. Please use lots of space.

It is very important that you show clearly any work you had to do to get your answers. Just writing the answer down with no work shown is not enough. All 15 questions are worth 2 points each. Hand in your solutions next week only.

Do these first 10 questions and *check that your answers match the solutions on page 3*. If you don't get the same answers then look at your notes or the book or ask me. Only do the last five questions when you are sure you understand the first ten.

(1) Find the domain and range of

$$(a) \quad f(x) = x^2 - 2, \qquad (b) \quad g(x) = \frac{1}{x+6}.$$

(2) Use a calculator to numerically estimate

$$\lim_{x \to 0} \frac{\tan x}{x}$$

by substituting the x values ± 0.1 and ± 0.01. (Make sure you are in radians mode.)

(3) Give the definition of the derivative of a function $f(x)$ at $x = a$.

(4) Using the formulas for differentiation, compute

$$(a) \quad \frac{d}{dx} x^{20}, \qquad (b) \quad \frac{d}{dx} \frac{5}{x}, \qquad (c) \quad \frac{d}{dx}(2x + \cos x - 3\tan x).$$

(5) Using the formulas for differentiation, find

$$(a) \quad \frac{d}{dx}\big(\sin(x)\cos(x)\big), \qquad (b) \quad \frac{d}{dx}\sin(\cos(x)), \qquad (c) \quad \frac{d}{dx}\sqrt{x}\tan^2(x).$$

(6) Find the most general antiderivatives of

$$(a) \quad x^8, \qquad (b) \quad 8 + 3\sin(x).$$

(7) Estimate the area between the curve $y = x^3$ and the x-axis for x between 0 and 3 using 3 rectangles of width 1 each. So the first rectangle has base $[0, 1]$, the second has base $[1, 2]$ and the third $[2, 3]$. Use the values of x^3 at the midpoints of these bases to find the rectangle heights. See the Midpoint Rule in the text for more information.

(8) Find the actual value of the area in the previous question by computing

$$\int_0^3 x^3\,dx.$$

(Find an antiderivative of x^3 and use Part 2 of the Fundamental Theorem of Calculus.)

(9) Using antiderivatives, compute

$$(a)\quad \int_0^{\pi/2} \cos(x)\,dx, \qquad (b)\quad \int_{-2}^{3} (x^4+1)\,dx.$$

(10) Use the Substitution Rule to calculate

$$\int_0^3 2x\sqrt{x^2+16}\,dx.$$

Five more questions. Show clearly all your working out and reasoning.

(11) Find the domain and range of

$$(a)\quad f(x)=\frac{x^3}{x}, \qquad (b)\quad g(x)=\tan(x).$$

(12) Using the formulas for differentiation, find

$$(a)\quad \frac{d}{dx}\big(6x^6-x^2/2\big), \qquad (b)\quad \frac{d}{dx}\big(x\sin^2(x)\big).$$

(13) Find the most general antiderivative of: $4x^9+\cos(x)$

(14) Using antiderivatives, compute

$$(a)\quad \int_0^{2\pi} \sin(t)\,dt, \qquad (b)\quad \int_{-3}^{4} (t^3+t^2)\,dt.$$

(15) Use the Substitution Rule to calculate

$$\int_0^{\pi^{1/3}} x^2\sin(x^3)\,dx.$$

Do more questions from chapters 1-4 in the book to make yourself familiar with the material in Calculus I. There are good review questions at the end of each chapter.

Answers to questions (1)-(10) on next page

Answers to questions (1)-(10):

(1) (a) Domain of f is $\mathbb{R}$, Range of f is $[-2, \infty)$.
(b) Domain of g is all of $\mathbb{R}$ except -6, Range of g is all of $\mathbb{R}$ except 0.

(2) The limit is 1.

(3) Find this definition in your class notes or the book.

(4)

$$(a) \quad 20x^{19}, \qquad (b) \quad -5/x^2, \qquad (c) \quad 2 - \sin(x) - 3\sec^2(x).$$

(5)

$$(a) \ \cos^2(x) - \sin^2(x), \qquad (b) \ -\cos(\cos(x))\sin(x), \qquad (c) \ \frac{\tan^2(x)}{2\sqrt{x}} + 2\sqrt{x}\sec^2(x)\tan(x).$$

(6) (a) $x^9/9 + C$, (b) $8x - 3\cos(x) + C$.

(7) This approximation for the area is $153/8 = 19.125$.

(8) $81/4 = 21.25$

(9) (a) 1, (b) 60.

(10) $122/3$