**Math 06, Homework 12 on graphing, identities**
**due Mon, Dec 9 at the start of class.**

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Write all your answers on a separate sheet. It is very important that you show clearly any work you had to do to get the answer. These first eight questions are 1 point each. Make sure your answers match the solutions on page 2.

**(1)** For $y = 4\sin x$, sketch the graph for $0 \le x \le 2\pi$ and give the amplitude.

**(2)** Complete this table of values for $y = -3\cos x$

| $x$ | $\cos x$ | $-3\cos x$ |
|---|---|---|
| $0$ | | |
| $\pi/2$ | | |
| $\pi$ | | |
| $3\pi/2$ | | |
| $2\pi$ | | |

**(3)** For $y = -3\cos x$, sketch the graph for $0 \le x \le 2\pi$ and give the amplitude.

**(4)** For $y = \dfrac{1}{3}\sin x$, sketch the graph for $-2\pi \le x \le 2\pi$ and give the amplitude.

**(5)** Find the equation of the sine or cosine function which fits this information: The function has a maximum value of $10$ at $x = 0$. It then reaches a minimum value of $-10$ at $x = \pi$. The function attains its maximum value again at $x = 2\pi$.

**(6)** Explain if this equation is an identity or not:

$$\cos\theta + \sin\theta = 1$$

**(7)** Verify the identity:

$$\sin\theta\sec\theta = \tan\theta$$

**(8)** Verify the identity:

$$(\sec t + 1)(\sec t - 1) = \tan^2 t$$

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These next seven questions are 4 points each. Show clearly all your working out and reasoning.

**(9)** Complete this table of values for $y = -\sin x$

| $x$ | $y = -\sin x$ |
|---|---|
| $0$ | |
| $\pi/2$ | |
| $\pi$ | |
| $3\pi/2$ | |
| $2\pi$ | |

**(10)** For $y = -\sin x$, sketch the graph for $0 \le x \le 2\pi$ and give the amplitude.

**(11)** For $y = \dfrac{3}{4}\cos x$, sketch the graph for $-\pi \le x \le 3\pi$ and give the amplitude.

**(12)** Find the equation of the sine or cosine function which fits this information: The function has a maximum value of 6 at $x = \pi/2$. It then reaches a minimum value of $-6$ at $x = 3\pi/2$. The function attains its maximum value again at $x = 5\pi/2$.

**(13)** Explain if this equation is an identity or not:

$$\sin\theta + \sec\theta = \tan\theta$$

**(14)** Verify the identity:

$$\sin\alpha = \frac{\tan\alpha \cot\alpha}{\csc\alpha}$$

**(15)** Verify the identity:

$$\frac{\csc\theta}{\cot\theta + \tan\theta} = \cos\theta$$

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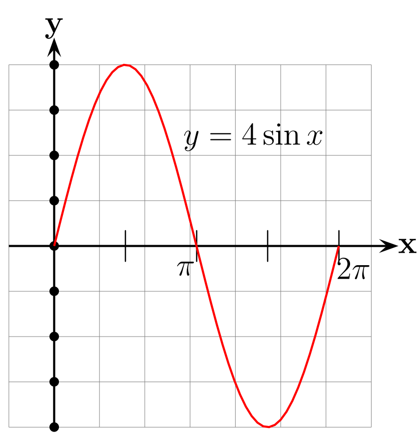
**Answers to questions (1)-(8):**

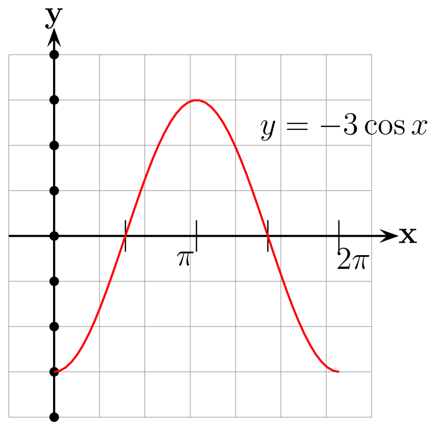
*(1)* The amplitude of $y = 4\sin x$ is $4$.



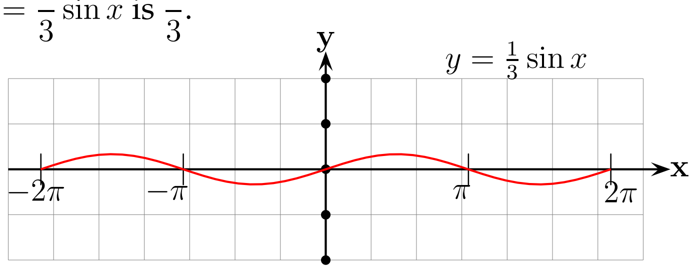
(2)

| $x$ | $\cos x$ | $-3\cos x$ |
|---|---|---|
| 0 | 1 | -3 |
| $\pi/2$ | 0 | 0 |
| $\pi$ | -1 | 3 |
| $3\pi/2$ | 0 | 0 |
| $2\pi$ | 1 | -3 |

(3) The amplitude of $y = -3\cos x$ is 3.




(4) The amplitude of $y = \frac{1}{3}\sin x$ is $\frac{1}{3}$.




(5) $y = 10\cos x$

(6) For $\theta = \pi$ we have

$$\cos\theta + \sin\theta = \cos\pi + \sin\pi = -1 + 0 = -1$$

and so $\cos\theta + \sin\theta = 1$ is not an identity since it's not true for all numbers $\theta$.

(7) Starting with the left side:

$$\begin{aligned} \sin\theta\sec\theta &= \sin\theta\frac{1}{\cos\theta} && \text{(from definition of } \sec\theta) \\ &= \frac{\sin\theta}{\cos\theta} && \text{(algebra)} \\ &= \tan\theta && \text{(from definition of } \tan\theta). \end{aligned}$$

This proves the identity $\sin\theta\sec\theta = \tan\theta$.

(8) Starting with the left side:

$$\begin{aligned} (\sec t + 1)(\sec t - 1) &= \sec^2 t - 1 && \text{(by multiplying out)} \\ &= \tan^2 t && \text{(by a Pythagorean identity)}. \end{aligned}$$

This proves the identity $(\sec t + 1)(\sec t - 1) = \tan^2 t$.